Multi-city SIR Epidemic Model

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Simple SIR Model

$$dS = -\lambda S$$

$$dI = \lambda S - \alpha I$$

$$dR = \alpha I$$

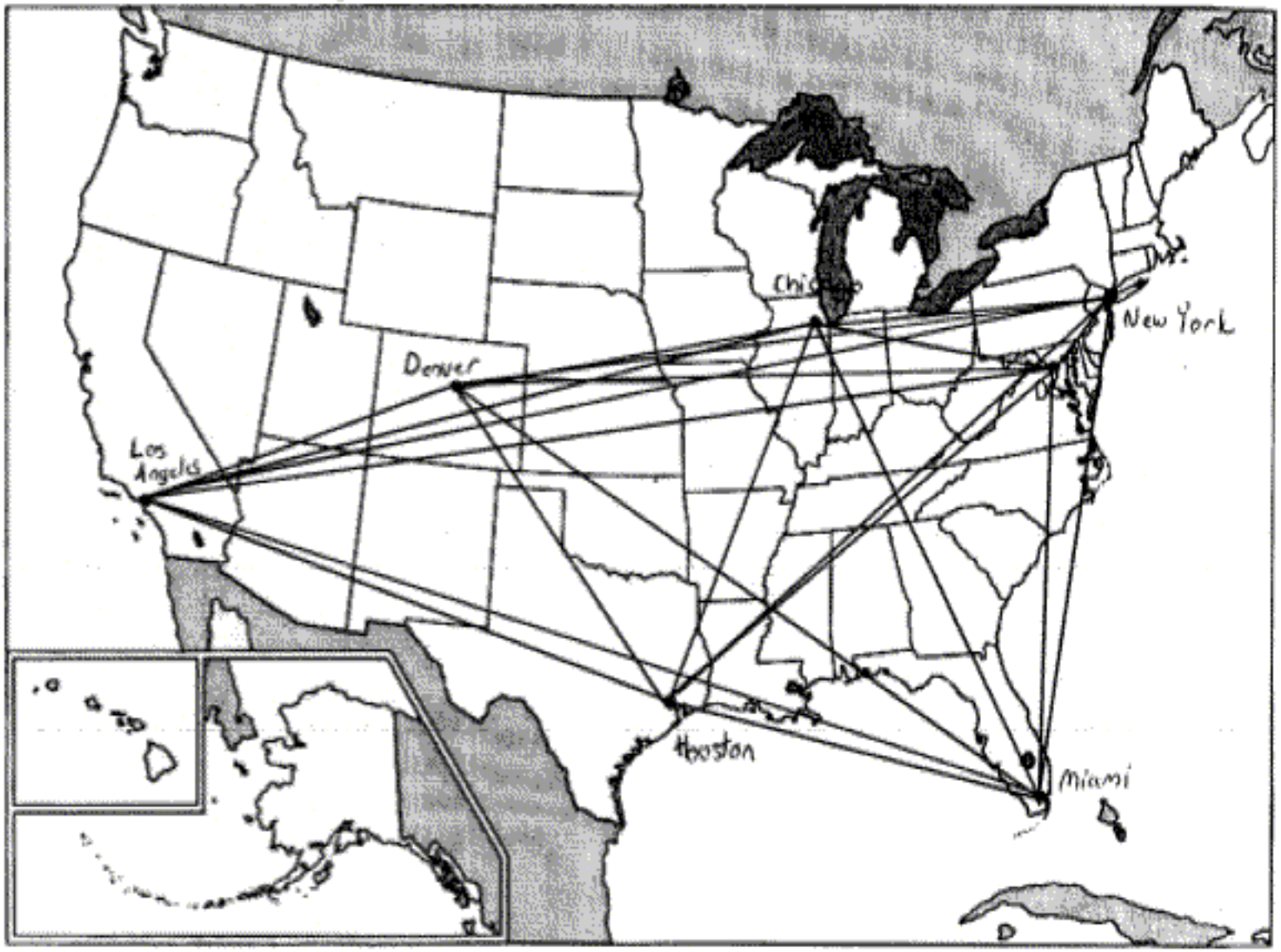
where

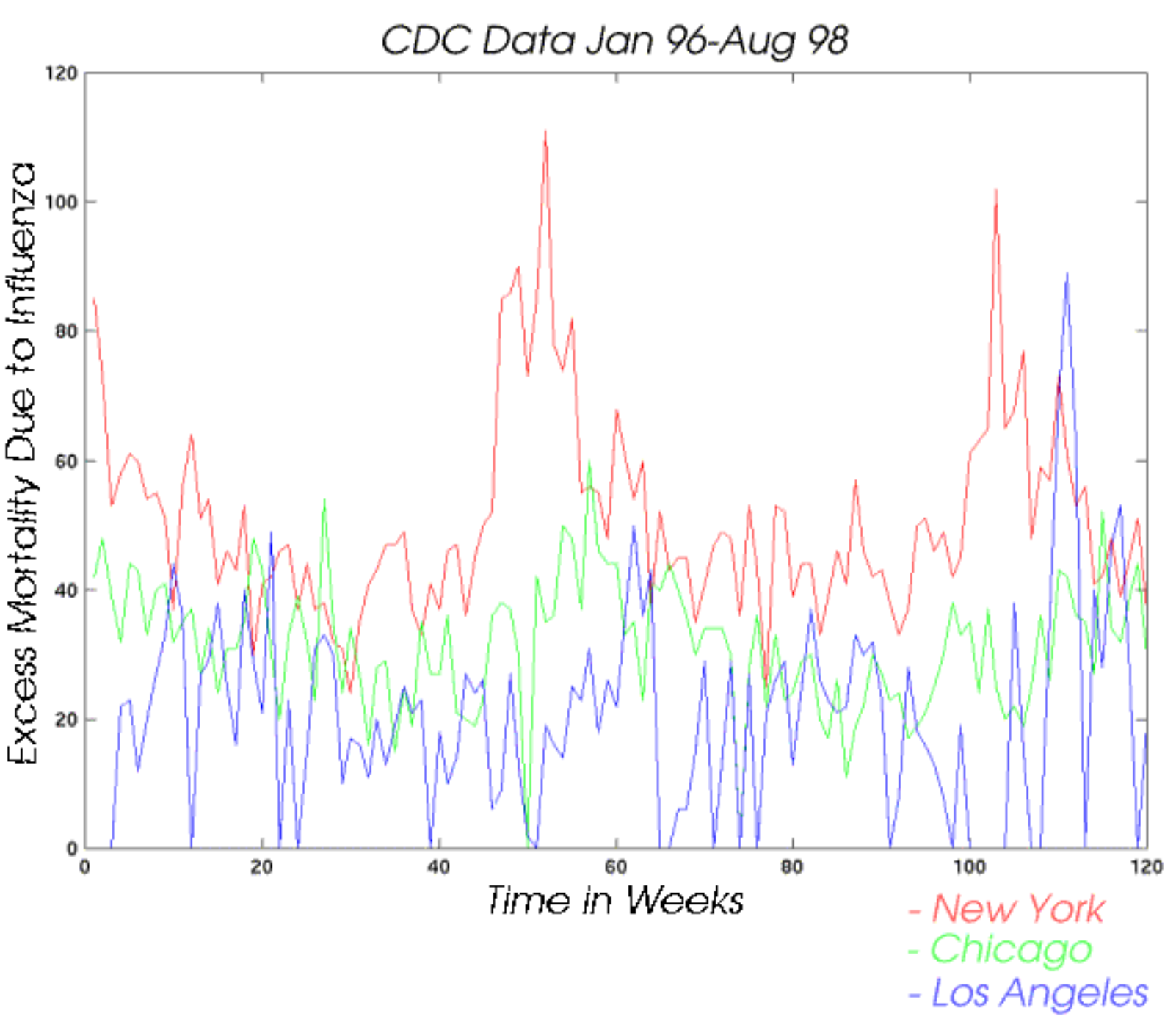
$\lambda = \beta\rho(I/N)$

ρ = # of contacts per day

β = infection rate per contact

α = recovery rate per day


Chicago
New York
Denver
Los Angeles
Houston
Miami


CDC Data Jan 96-Aug 98
Excess Mortality Due to Influenza
Time in Weeks
0
20
40
60
80
100
120
- New York
- Chicago
- Los Angeles

Two City SIR Model

$$dS_1 = -\lambda S_1 - \delta_{AB}(S_1/N_1) + \delta_{BA}(S_2/N_2) + \mu(S_{o1}-S_1)$$

$$dI_1 = \lambda S_1 - \alpha I_1 - \delta_{AB}(I_1/N_1) + \delta_{BA}(I_2/N_2) - \mu(I_1)$$

$$dR_1 = \alpha I_1 - \delta_{AB}(R_1/N_1) + \delta_{BA}(R_2/N_2) - \mu(R_1)$$

$$dS_2 = -\lambda S_2 - \delta_{BA}(S_2/N_2) + \delta_{AB}(S_1/N_1) + \mu(S_{o2}-S_2)$$

$$dI_2 = \lambda S_2 - \alpha I_2 - \delta_{BA}(I_2/N_2) + \delta_{AB}(I_1/N_1) - \mu(I_2)$$

$$dR_2 = \alpha I_2 - \delta_{BA}(R_2/N_2) + \delta_{AB}(R_1/N_1) - \mu(R_2)$$

where

δ_{AB} = # of people traveling from city A to city B

δ_{BA} = # of people traveling from city B to city A

μ = natural birth/death rate in the population

Multiple-city SIR Model

$$dS_i = -\lambda S_i - \delta_{out}(S_i/N_i) + \delta_{in}(S_2/N_2) + \mu(S_{oi}-S_i)$$

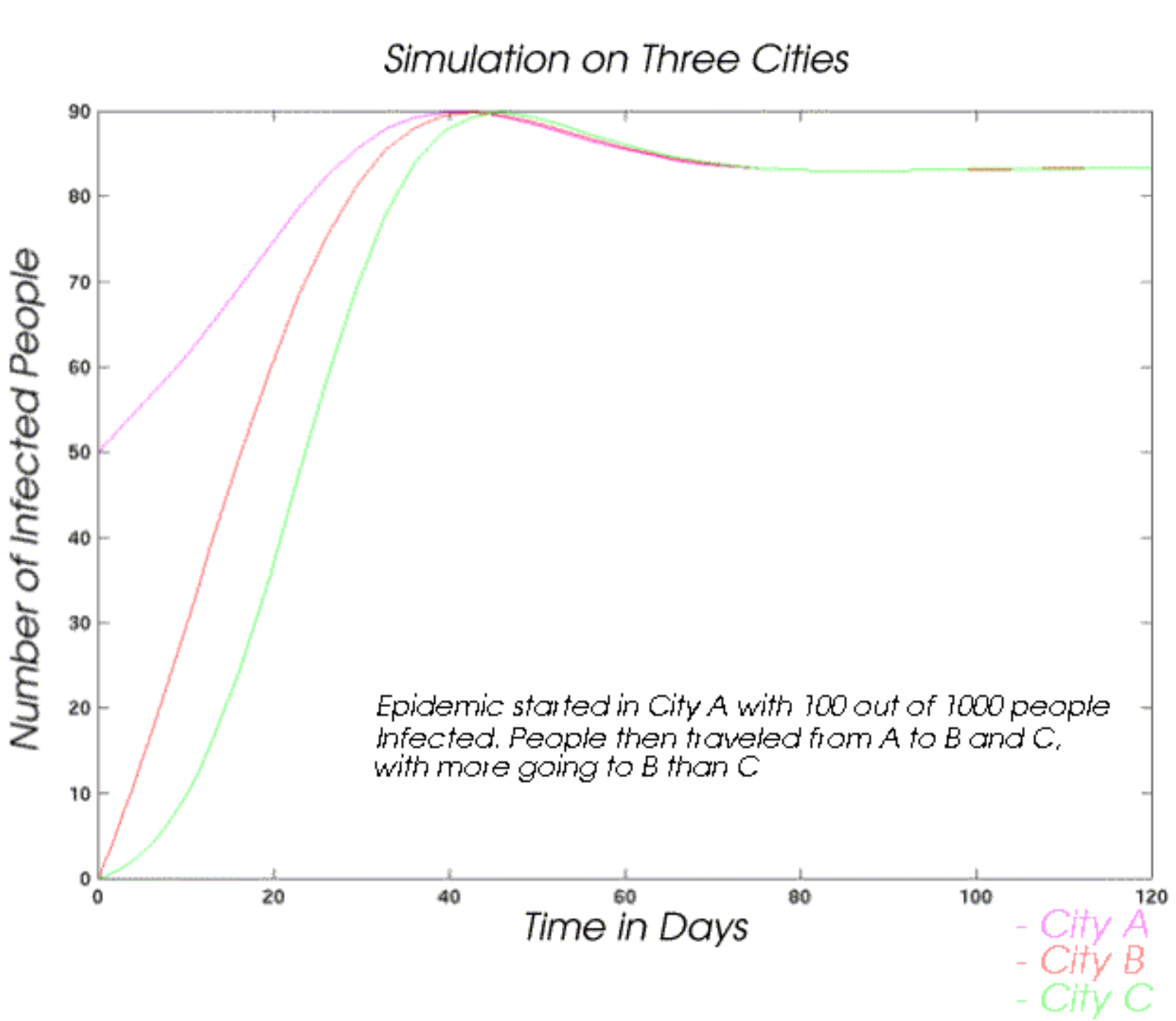
$$dI_i = \lambda S_i - \alpha I_i - \delta_{out}(I_i/N_i) + \delta_{in}(I_2/N_2) - \mu(I_i)$$

$$dR_i = \alpha I_i - \delta_{out}(R_i/N_i) + \delta_{in}(R_2/N_2) - \mu(R_i)$$

where

δ_{out} = total number of people leaving city i for any other city

δ_{in} = total number of people entering city i from any other city


Simulation on Three Cities
Number of Infected People
Time in Days
0
10
20
30
40
50
60
70
80
90
0
20
40
60
80
100
120
Epidemic started in City A with 100 out of 1000 people
Infected. People then traveled from A to B and C,
with more going to B than C
- City A
- City B
- City C

Work in Progress

- Create sinusoidal forcing term
- Find out how much migration between cities is necessary for the epidemic to spread in the model
- Run model on CDC data with flight information to see how model results compare with real data